

Influence of quantum degeneracy and regeneration on the performance of Bose-Stirling refrigeration-cycles operated in different temperature regions

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Abstract

The Stirling refrigeration cycle using an ideal Bose-gas as the working substance is called the Bose-Stirling refrigeration cycle, which is different from other thermodynamic cycles such as the Carnot cycle, Ericsson cycle, Brayton cycle, Otto cycle, Diesel cycle and Atkinson cycle working with an ideal Bose gas and may be operated across the critical temperature of Bose–Einstein condensation of the Bose system. The performance of the cycle is investigated, based on the equation of state of an ideal Bose gas. The inherent regenerative losses of the cycle are considered and the coefficient of performance and the amount of refrigeration of the cycle are calculated. The results obtained here are compared with those derived from the classical Stirling refrigeration cycle, using an ideal gas as the working substance. The influence of quantum degeneracy and inherent regenerative losses on the performance of the Bose Stirling refrigeration cycle operated in different temperature regions is discussed in detail, and consequently, general performance characteristics of the cycle are revealed.

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Keywords: Stirling refrigeration cycle; Bose gas; Quantum degeneracy; Regenerative loss; Temperature region; Performance analysis

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Nomenclature

C_V	heat capacity at constant volume (J)
$F(z)$	correction function
g	weight factor
$g_f(z)$	Bose function
h	Planck's constant (J s)
k	Boltzmann's constant (J/K)
m	rest mass of a particle (kg)
N	total number of particles
N_0	particle number in the ground state
n	number density (m^{-3})
P	pressure (Pa)
Q_1	amount of heat exchanged between the working substance and the regenerator for a constant V_H process (J)
Q_2	amount of heat exchanged between the working substance and the regenerator for a constant V_L process (J)
Q_H	amount of heat exchanged between the working substance and the heat reservoir at temperature T_H during the isothermal process (J)
Q_L	amount of heat exchanged between the working substance and the heat reservoir at temperature T_L during the isothermal process (J)
R_e	relative coefficient of performance
R_{QL}	relative refrigeration load
r_v	volume ratio of two isochoric processes (J)
S	entropy of the working substance (J/K)
T	temperature of the working substance (K)
T_C	condensation temperature (K)
T_E	temperature at which the heat capacities at two constant volumes are equal to each other (K)
T_H	temperature of the high-temperature heat-reservoir (K)
T_L	temperature of the low-temperature heat-reservoir (K)
U	internal energy (J)
V	volume (m^3)
V_H	maximum volume of the gas (m^3)
V_L	minimum volume of the gas (m^3)
v	average volume that an ideal Bose particle occupies (m^3)
v_H	maximum average volume of an ideal Bose particle (m^3)
v_L	minimum average volume of an ideal Bose particle (m^3)
W	work input per cycle (J)
z	fugacity of the gas
ΔQ	regenerative loss (J)
ε	coefficient of performance
λ	mean thermal wavelength (m)

μ	chemical potential of the gas (J)
τ	temperature ratio of two heat-reservoirs
$\Gamma(l)$	Gamma function

1. Introduction

It is well known that the performance of a thermodynamic cycle using a classical ideal gas as the working substance can be found directly from thermodynamics textbooks. However, when the gas temperature is low enough or density is high enough, the gas will deviate from classical gas behavior and quantum degeneracy of the gas will become important [1–3]. It is thus clear that one needs to consider the influence of quantum degeneracy on the performance of a thermodynamic cycle at low temperatures. Recently, several authors have analyzed the effect of quantum degeneracy on the performance of the engine and refrigeration cycles working with an ideal Bose or Fermi gas [4–9], based on quantum statistical mechanics. The influence of several factors on the performance of these cycles has been analyzed and many meaningful conclusions have been obtained.

Like classical thermodynamic cycles [10,11], quantum thermodynamic cycles using an ideal Bose gas as the working substance may have different cycle models such as the Carnot cycle, Ericsson cycle, Stirling cycle, Brayton cycle, Otto cycle, Diesel cycle, Atkinson cycle, and so on. It has been proved [12], according to the properties of an ideal Bose gas, that some important thermodynamic processes such as a constant pressure or an adiabatic process cannot be carried out from the temperature region of $T > T_c$ to that of $T < T_c$, where T is the temperature of the Bose system and T_c is the critical temperature of Bose–Einstein condensation (BEC) of the Bose system. Consequently, the Carnot cycle, Brayton cycle, Otto cycle, Ericsson cycle, Diesel cycle and Atkinson cycle working with an ideal Bose gas can only be operated in the temperature region above T_c because these thermodynamic cycles include the adiabatic or/and isobaric process. The Stirling cycle is also one of the important thermodynamic cycles. However, unlike the thermodynamic cycles mentioned above, the Stirling cycle consisting of two isochoric and two isothermal processes does not include the adiabatic or isobaric process and can be operated across the critical temperature of BEC of the Bose system when an ideal Bose gas is used as the working substance. Thus, the Stirling cycle working with an ideal Bose gas can be operated in different temperature regions and will have some novel performance characteristics which need to be studied further.

In the present paper, we will study the performance of the Stirling refrigeration cycle using an ideal Bose gas as the working substance. First, the general expressions for several important parameters such as the coefficient of performance, amount of refrigeration and work input per cycle are derived, based on the equation of state of an ideal Bose gas; secondly, the influence of quantum degeneracy and inherent regenerative losses on the performance of the cycle are analyzed; Finally, some special cases are discussed in detail, so that the general performance characteristics of the

Bose Stirling refrigeration cycle operated in the various different temperature regions are revealed.

2. The Bose-Stirling refrigeration-cycle model

The Stirling refrigeration cycle working with an ideal Bose gas is called the Bose-Stirling refrigeration cycle, which is composed of two isothermal and two isochoric processes. It is operated between the heat sink at temperature T_H and the cooled space at temperature T_L . In order to improve the performance of the cycle, a regenerator is used in the isochoric processes. The temperature–entropy diagram of the cycle is shown in Fig. 1, where Q_L and Q_H are the amounts of heat exchanged between the working substance and the heat reservoirs at temperatures T_L and T_H during the isothermal processes, Q_1 and Q_2 are the amounts of heat exchanged between the working substance and the regenerator during the isochoric processes, and V_L and V_H are the minimum and maximum volumes of the gas system. For the sake of convenience, all the heats Q_L , Q_H , Q_1 and Q_2 are selected to be positive.

For an ideal Bose gas, the expressions of its pressure and number density of particles are given by [1–3]

$$P = gkT\lambda^{-3}g_{5/2}(z) \quad (1)$$

and

$$n = (N - N_0)/V = g\lambda^{-3}g_{3/2}(z), \quad (2)$$

respectively, where g is the number of possible spin orientations of a gas particle, k is the Boltzmann constant, T , N , N_0 , and V are, respectively, the gas temperature, total

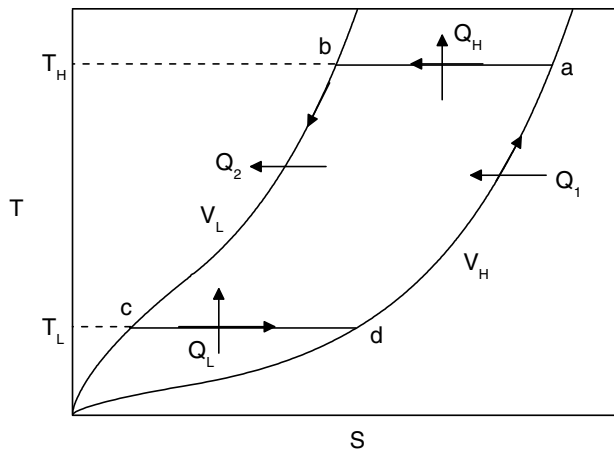


Fig. 1. The temperature–entropy diagram of a Bose-Stirling refrigeration-cycle.

number of particles, number of particles in the ground state, and volume, $z = \exp(\mu/kT)$ and $\lambda = h(2m\pi kT)^{-1/2}$ are, respectively, the fugacity of the gas and the mean thermal wavelength of particles, μ is the chemical potential of the gas, h is the Planck constant, m is the rest mass of a particle

$$g_l(z) = \frac{1}{\Gamma(l)} \int_0^\infty \frac{x^{l-1} dx}{z^{-1}e^x - 1}, \quad (3)$$

is called the Bose function, and $\Gamma(l)$ is the Gamma function. When $T \geq T_c$, the particle population of the ground state is macroscopically negligible compared to the total particle number of the ideal Bose gas system [1–3]. Consequently, it is seen from Eq. (2) that for an ideal Bose gas, the number density of particles, n , should be equal to the number density of particles that have energies larger than the energy of the ground state. Using Eqs. (1) and (2), one can find that the internal energy and entropy of an ideal Bose gas system are, respectively, given by [1]

$$U = \frac{3}{2}PV = \begin{cases} \frac{3}{2}NkTg_{5/2}(z)/g_{3/2}(z), & T \geq T_c, \\ \frac{3}{2}(N - N_0)kT \frac{\zeta(5/2)}{\zeta(3/2)} = \frac{3}{2} \frac{\zeta(5/2)}{\zeta(3/2)} NkT \left(\frac{T}{T_c}\right)^{3/2}, & T < T_c, \end{cases} \quad (4)$$

and

$$S = \begin{cases} Nk \left[\frac{5}{2} g_{5/2}(z)/g_{3/2}(z) - \ln z \right], & T \geq T_c, \\ \frac{5}{2}(N - N_0)k \frac{\zeta(5/2)}{\zeta(3/2)} = \frac{5}{2} \frac{\zeta(5/2)}{\zeta(3/2)} NkT \left(\frac{T}{T_c}\right)^{3/2}, & T < T_c, \end{cases} \quad (5)$$

where $\zeta(5/2) = 1.341$, $\zeta(3/2) = 2.612$, $T_c = \frac{h^2}{2\pi mk[v\zeta(3/2)]^{2/3}}$ and v is the specific volume.

From Eq. (4), one can prove that the heat capacity at constant volume can be expressed as

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{3}{2}Nk \frac{d}{dT} [TF(T, v)] = \begin{cases} \frac{15}{4}Nk \left[\frac{g_{5/2}(z)}{g_{3/2}(z)} - \frac{3}{5} \frac{g_{3/2}(z)}{g_{1/2}(z)} \right], & T \geq T_c, \\ \frac{15}{4}Nk \frac{\zeta(5/2)}{\zeta(3/2)} \left(\frac{T}{T_c}\right)^{3/2}, & T < T_c, \end{cases} \quad (6)$$

where

$$F(T, v) = \begin{cases} g_{5/2}[z(T, v)]/g_{3/2}[z(T, v)], & T \geq T_c, \\ [\zeta(5/2)/\zeta(3/2)][T/T_c]^{3/2}, & T < T_c. \end{cases}$$

Using Eqs. (5) and (6), the amounts of heat exchanged in an isothermal and an isochoric process are, respectively, given by

$$\begin{aligned} Q_{if}^T &= \int_{S_i}^{S_f} T dS = T(S_f - S_i) \\ &= NkT \left\{ \frac{5}{2} [F(T, v_f) - F(T, v_i)] - [\ln z(T, v_f) - \ln z(T, v_i)] \right\} \end{aligned} \quad (7)$$

and

$$Q_{if}^v = \int_{T_i}^{T_f} C_V(T, v) dT = \frac{3}{2} Nk [T_f F(T_f, v) - T_i F(T_i, v)], \quad (8)$$

where the indices *i* and *f* refer to the initial and final states, respectively.

3. Regenerative characteristics

From Eq. (8), one can find that the net amount of heat transfer between the working substance and the regenerator during two isochoric processes is determined by

$$\begin{aligned} \Delta Q &= Q_2 - Q_1 = \int_{T_L}^{T_H} C_V(T, v_L) dT - \int_{T_L}^{T_H} C_V(T, v_H) dT \\ &= \frac{3}{2} Nk \{ T_H [F(T_H, v_L) - F(T_H, v_H)] - T_L [F(T_L, v_L) - F(T_L, v_H)] \}. \end{aligned} \quad (9)$$

Using Eq. (6), we can plot the C_V/Nk versus T characteristic curves, as shown in Fig. 2. It is clearly seen from Fig. 2 that $C_V(T, v_L) > C_V(T, v_H)$ when $T > T_E$ and $C_V(T, v_H) > C_V(T, v_L)$ when $T < T_E$, where T_E is the temperature at which the heat capacities at two constant volumes are equal to each other, i.e., $C_V(T_E, v_L) = C_V(T_E, v_H)$. Consequently, it can be seen from Eq. (9) that there are two possible cases: (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$. When $\Delta Q > 0$, the amount of heat Q_2 flowing into the regenerator in the small constant-volume regenerative process is larger than that of Q_1 flowing from the regenerator in the large constant-volume regenerative process. The redundant heat in the regenerator per cycle must be released to the cold reservoir in a timely manner. This results in the reduction of the amount of refrigeration from Q_L to Q'_L . If not, the temperature of the regenerator would be changed

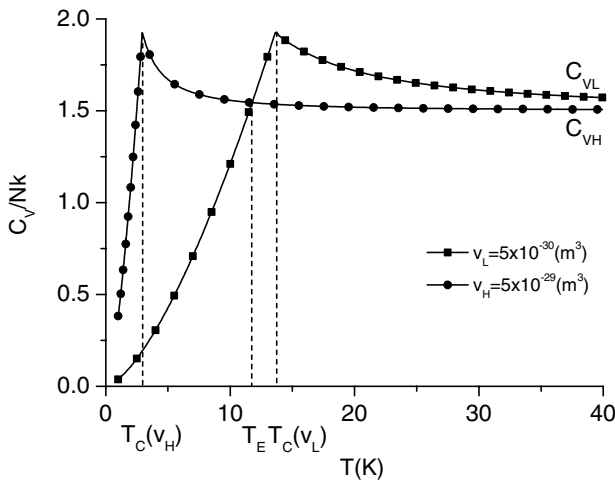


Fig. 2. The curves of C_V varying with T for two different specific volumes.

such that the regenerator would not be operated normally. Similarly, when $\Delta Q < 0$, the amount of heat Q_2 flowing into the regenerator in the small constant-volume regenerative process is smaller than that of Q_1 flowing from the regenerator in the large constant-volume regenerative process. The inadequate heat in the regenerator per cycle must be compensated from the hot reservoir in a timely manner, while the amount of refrigeration Q_L is unvarying.

According to the regenerative characteristics mentioned above, the unified expression for the amount of refrigeration per cycle can be given by

$$\begin{aligned} Q'_L &= Q_L - \delta \Delta Q \\ &= NkT_L \left\{ \frac{5}{2} [F(T_L, v_H) - F(T_L, v_L)] - [\ln z(T_L, v_H) - \ln z(T_L, v_L)] \right\} \\ &\quad - \delta \frac{3}{2} Nk \{ T_H [F(T_H, v_L) - F(T_H, v_H)] - T_L [F(T_L, v_L) - F(T_L, v_H)] \}, \end{aligned} \quad (10)$$

where $\delta = 1$ when $\Delta Q > 0$ and $\delta = 0$ when $\Delta Q < 0$.

4. Expressions for several important parameters

Besides the amount of refrigeration, the coefficient of performance and work input are also the important performance parameters of the cycle. Using Eqs. (7), (8) and (10), one can find that the work input per cycle and coefficient of performance may be, respectively, expressed as

$$\begin{aligned} W &= Q_H - Q_L + Q_2 - Q_1 \\ &= NkT_H [F(T_H, v_H) - F(T_H, v_L) + \ln z(T_H, v_L) - \ln z(T_H, v_H)] \\ &\quad + NkT_H [F(T_H, v_H) - F(T_H, v_L) + \ln z(T_H, v_L) - \ln z(T_H, v_H)] \end{aligned} \quad (11)$$

and

$$\varepsilon = \frac{T_L \left\{ \frac{5}{2} [F(z_d) - F(z_c)] - \ln(z_d/z_c) \right\} - \delta \frac{3}{2} \{ T_H [F(z_b) - F(z_a)] - T_L [F(z_c) - F(z_d)] \}}{T_H [F(z_a) - F(z_b) + \ln(z_b/z_a)] + T_L [F(z_c) - F(z_d) + \ln(z_d/z_c)]}, \quad (12)$$

where z_a , z_b , z_c and z_d are the fugacity of the gas in state points a, b, c and d shown in Fig. 1.

In addition, in order to compare the performance of a Bose-Stirling refrigeration cycle with that of a classical Stirling refrigeration cycle using a classical ideal gas as the working substance, we introduce the relative amount of refrigeration and relative coefficient of performance as follows:

$$\begin{aligned} R_{QL} &= \frac{Q'_L}{Q_L^c} \\ &= \frac{\frac{5}{2} [F(z_d) - F(z_c)] - \ln(z_d/z_c) - \delta \frac{3}{2} \{ \tau [F(z_b) - F(z_a)] - [F(z_c) - F(z_d)] \}}{\ln r_v} \end{aligned} \quad (13)$$

and

$$R_\varepsilon = \frac{\varepsilon}{\varepsilon_c} = \frac{(\tau - 1) \left\{ \frac{\delta}{2} [F(z_d) - F(z_c)] - \ln(z_d/z_c) - \delta \frac{3}{2} \{ \tau [F(z_b) - F(z_a)] + F(z_d) - F(z_c) \} \right\}}{\tau [F(z_a) - F(z_b) + \ln(z_b/z_a)] + F(z_c) - F(z_d) + \ln(z_d/z_c)}, \quad (14)$$

where $Q_L^c = NkT_L \ln r_v$ and $\varepsilon_c = T_L/(T_H - T_L)$ are, respectively, the amount of refrigeration and coefficient of performance of a classical Stirling refrigeration cycle with perfect regeneration, and $\tau = T_H/T_L$ and $r_v = v_H/v_L$ are, respectively, the temperature ratio of the reservoirs and the volume ratio of two constant-volume processes.

Using the above equations, we can discuss the performance characteristics of the cycle operated in the various different temperature-regions.

5. General performance characteristics

It is clearly seen from Eqs. (9)–(12) that, for a Bose-Stirling refrigeration cycle, the amount of refrigeration, work input and coefficient of performance are, in general, dependent on the regeneration characteristic, temperature, volume and other parameters, while the net heat addition to or rejection from the regenerator depends on the magnitude order of T_H , T_L and T_E except for depending on the temperature and volume. It is obvious that there are different performance characteristics for the Bose-Stirling refrigeration cycle operated in the different temperature regions, which will be discussed below.

• When $T_H > T_L > T_E$, $C_V(T, v_H) < C_V(T, v_L)$ and $\Delta Q > 0$. For the case of $T_L > T_C(v_L)$, Eqs. (12)–(14) can be, respectively, written as

$$\varepsilon = \frac{T_L[F(z_d) - F(z_c) - \ln(z_d/z_c)] + (3/2)T_H[F(z_a) - F(z_b)]}{T_H[F(z_a) - F(z_b) + \ln(z_b/z_a)] - T_L[F(z_d) - F(z_c) - \ln(z_d/z_c)]}, \quad (15)$$

$$R_{QL} = \frac{F(z_d) - F(z_c) - \ln(z_d/z_c) + (3/2)\tau[F(z_a) - F(z_b)]}{\ln r_v}, \quad (16)$$

and

$$R_\varepsilon = \frac{(\tau - 1) \{ F(z_d) - F(z_c) - \ln(z_d/z_c) + (3/2)\tau[F(z_a) - F(z_b)] \}}{\tau[F(z_a) - F(z_b) + \ln(z_b/z_a)] + F(z_c) - F(z_d) + \ln(z_d/z_c)}, \quad (17)$$

while for the case of $T_c(v_L) > T_L$, Eqs. (12)–(14) can be further simplified as

$$\varepsilon = \frac{T_L[F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d] + (3/2)T_H[F(z_a) - F(z_b)]}{T_H[F(z_a) - F(z_b) + \ln(z_b/z_a)] - T_L[F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}, \quad (18)$$

$$R_{QL} = \frac{F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d + (3/2)\tau[F(z_a) - F(z_b)]}{\ln r_v}, \quad (19)$$

and

$$R_e = \frac{(\tau - 1) \left\{ F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d + (3/2)\tau[F(z_a) - F(z_b)] \right\}}{\tau[F(z_a) - F(z_b) + \ln(z_b/z_a)] + 0.5134(T_L/T_{CL})^{3/2} - F(z_d) + \ln(z_d)}, \quad (20)$$

where $T_{CL} = \frac{h^2}{2\pi m k [v_L \zeta(3/2)]^{2/3}}$.

Using Eqs. (1)–(3), (16) and (19) and choosing He_4 gas as the ideal Bose gas, one can plot the curves of R_{QL} varying with the temperature T_L of the cooled space for given v_L , τ and r_v , as shown in Fig. 3. It is clearly seen from Fig. 3 that when the temperature ratio of the heat reservoirs τ is small, the relative amount of refrigeration R_{QL} is a monotonically decreasing function of T_L , but it is always greater than unity; when the temperature ratio of the heat reservoirs τ is large, R_{QL} is a monotonically increasing function of T_L , but it is always smaller than unity. This implies that when τ is small, the amount of refrigeration of the Bose-Stirling refrigeration cycle is greater than that of the classical Stirling refrigeration cycle because the effect of quantum degeneracy on the amount of refrigeration is larger than that of regenerative losses; when τ is large, the amount of refrigeration of the Bose-Stirling refrigeration cycle is smaller than that of the classical Stirling refrigeration cycle because the effect of regenerative losses on the amount of refrigeration is larger than that of quantum degeneracy. In addition, Fig. 3 shows that there is a value of the temperature ratio τ for which the amount of refrigeration ratio R_{QL} is equal to unity. This indicates that the influence of quantum degeneracy and regenerative losses on the amount of refrigeration of the cycle counteract one another in this case, so that the amount of refrigeration of the Bose-Stirling refrigeration cycle is the same as that of the classical Stirling refrigeration cycle. It can be also seen, from Fig. 3, that when the temperature ratio τ is small, the larger the volume ratio r_v is, the smaller the rel-

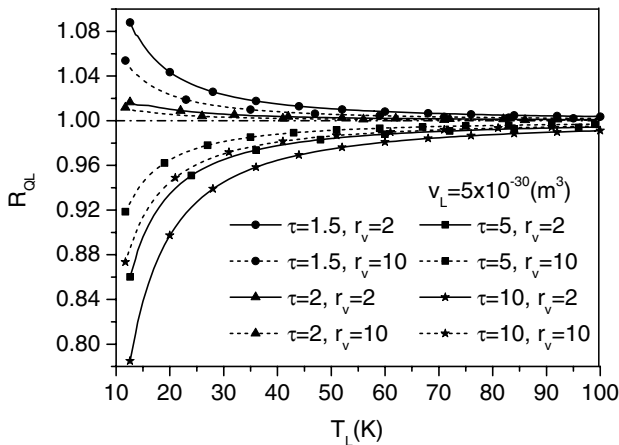


Fig. 3. The curves of R_{QL} varying with T_L in the temperature region of $T_H > T_L > T_E$ for some given values of v_L , r_v and τ .

ative amount of refrigeration R_{QL} ; when the temperature ratio τ is large, the larger the volume ratio r_v and the larger the relative amount of refrigeration R_{QL} . Obviously, R_{QL} tends to unity with the increase of T_L . This is an expected result because the quantum behavior of gas particles is negligible in this case.

Similarly, using Eqs. (1)–(3), (17) and (20) and choosing He_4 gas as the ideal Bose gas, we can plot the curves of R_e varying with the temperature T_L of the cooled space for given different parameter conditions, as shown in Fig. 4. It can be seen, from Fig. 4, that the relative coefficient of performance R_e is a monotonically increasing function of T_L for given v_L , τ and r_v , but it is always smaller than unity. This implies that the coefficient of performance of the Bose-Stirling refrigeration cycle is smaller than that of the classical Stirling refrigeration cycle and decreases as the quantum degeneracy of the working substance increases. The figure shows that for given T_L , v_L and r_v , the larger the temperature ratio τ , the smaller the relative coefficient of performance R_e ; for given T_L , v_L and τ , the larger the volume ratio r_v , the larger the R_e .

• When $T_H > T_c(v_L) > T_E > T_L > T_c(v_H)$, there may have two cases of $\Delta Q > 0$ and $\Delta Q < 0$. If $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT > \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$, $\Delta Q > 0$ and Eqs. (12)–(14) can be, respectively, expressed as

$$\varepsilon = \frac{T_L[F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d] + (3/2)T_H[F(z_a) - F(z_b)]}{T_H[F(z_a) - F(z_b) + \ln(z_b/z_a)] - T_L[F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}, \quad (21)$$

$$R_{QL} = \frac{F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d + (3/2)\tau[F(z_a) - F(z_b)]}{\ln(v_H/v_L)}, \quad (22)$$

and

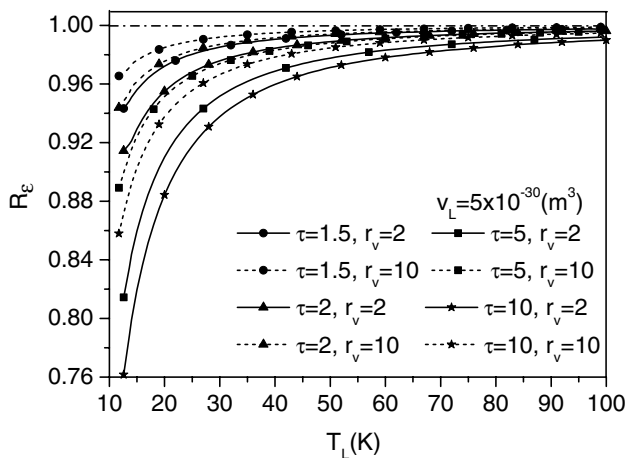


Fig. 4. The curves of R_e varying with T_L in the temperature region of $T_H > T_L > T_E$ for some given values of v_L , r_v and τ .

$$R_\varepsilon = \frac{(\tau - 1) \left\{ F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d + (3/2)\tau[F(z_a) - F(z_b)] \right\}}{\tau[F(z_a) - F(z_b) + \ln(z_b/z_a)] + 0.5134(T_L/T_{CL})^{3/2} - F(z_d) + \ln(z_d)}. \quad (23)$$

If $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT < \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$, $\Delta Q < 0$ and Eqs. (12)–(14) can be, respectively, expressed as

$$\varepsilon = \frac{T_L \left\{ (5/2)[F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d \right\}}{T_H[F(z_a) - F(z_b) + \ln(z_b/z_a)] - T_L[F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}, \quad (24)$$

$$R_{QL} = \frac{(5/2)[F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d}{\ln(v_H/v_L)}, \quad (25)$$

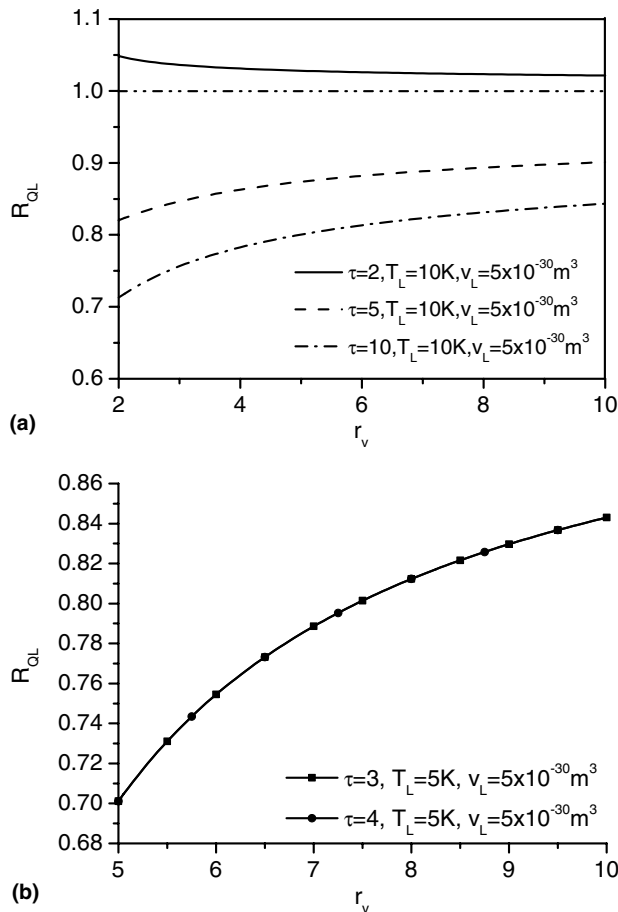


Fig. 5. The curves of R_{QL} varying with r_v in the temperature region of $T_H > T_c(v_L) > T_E > T_L > T_c(v_H)$ for some given values of v_L , T_L and τ . Curves are presented for (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$.

and

$$R_e = \frac{(\tau - 1) \left\{ (5/2) [F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d \right\}}{\tau [F(z_a) - F(z_b) + \ln(z_b/z_a)] - [F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}. \quad (26)$$

When $\Delta Q > 0$, the variations of the relative amount of refrigeration R_{QL} and the relative coefficient of performance R_e with the volume ratio r_v for given values of T_L , v_L and τ are shown in Figs. 5(a) and 6(a), respectively. It is seen, from the figures, that for given T_L and v_L , when the temperature ratio τ is small, R_{QL} is larger than unity and it is a monotonically decreasing function of r_v , while the coefficient of performance of the Bose-Stirling refrigeration cycle is smaller than that of the classical Stirling refrigeration cycle and it is a monotonically increasing function of r_v ; when the temperature ratio τ is large, both R_{QL} and R_e are always smaller than unity and both of them are monotonically increasing functions of r_v . For given T_L , v_L and r_v , the larger the temperature ratio τ , the smaller the R_{QL} and R_e .

When $\Delta Q < 0$, we can obtain the curves of both R_{QL} and R_e varying with the volume ratio r_v for given different parameter conditions, as shown in Figs. 5(b) and 6(b), respectively. It is seen from the curves in the figures that both R_{QL} and R_e are smaller than unity and they are monotonically increasing functions of r_v for given T_L , v_L and τ . This indicates that the amount of refrigeration and coefficient of performance of the Bose Stirling refrigeration cycle are smaller than those of the classical Stirling refrigeration cycle.

• When $T_H > T_c(v_L) > T_E > T_c(v_H) > T_L$, there may have two cases of $\Delta Q > 0$ and $\Delta Q < 0$. If $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT > \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$, $\Delta Q > 0$ and Eqs. (12)–(14) are, respectively, expressed as

$$\varepsilon = \frac{0.5134 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}] + (3/2) T_H [F(z_a) - F(z_b)]}{T_H [F(z_a) - F(z_b) + \ln(z_b/z_a)] - 0.5134 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}, \quad (27)$$

$$R_{QL} = \frac{0.5134 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}] + (3/2) \tau [F(z_a) - F(z_b)]}{\ln(v_H/v_L)}, \quad (28)$$

and

$$R_e = \frac{(\tau - 1) \left\{ 0.5134 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}] + (3/2) \tau [F(z_a) - F(z_b)] \right\}}{\tau [F(z_a) - F(z_b) + \ln(z_b/z_a)] - 0.5134 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}. \quad (29)$$

Similarly, if $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT < \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$, $\Delta Q < 0$ and Eqs. (12)–(14) can be, respectively, expressed as

$$\varepsilon = \frac{1.2835 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{T_H [F(z_a) - F(z_b) + \ln(z_b/z_a)] - 0.5134 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}, \quad (30)$$

$$R_{QL} = \frac{1.2835 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\ln(v_H/v_L)}, \quad (31)$$

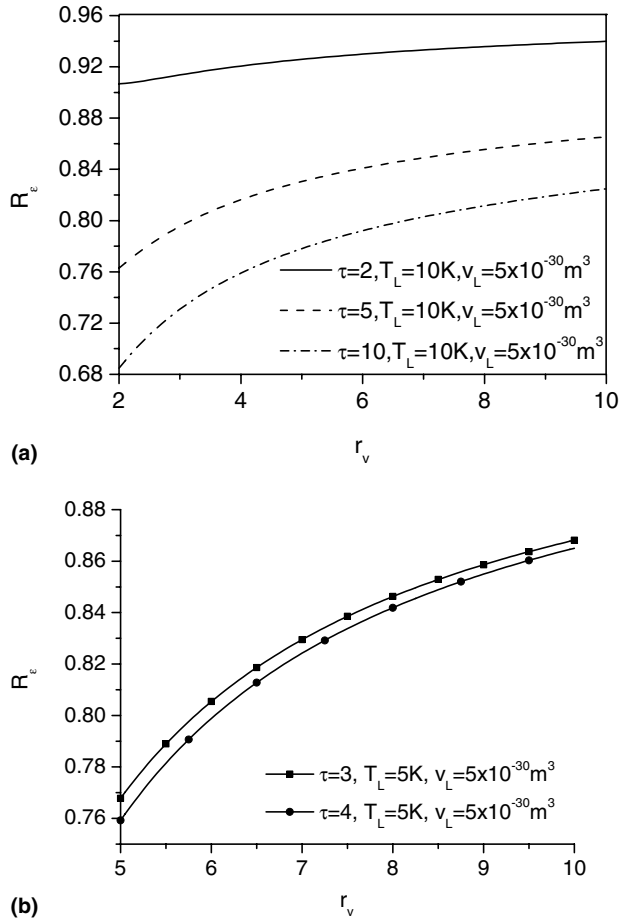


Fig. 6. The curves of R_e varying with r_v in the temperature region of $T_H > T_c(v_L) > T_E > T_L > T_c(v_H)$ for some given values of v_L , T_L and τ . Curves are presented for (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$.

and

$$R_e = \frac{1.2835(\tau - 1)[(T/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\tau[F(z_a) - F(z_b) + \ln(z_b/z_a)] - 0.5134[(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}, \quad (32)$$

where $T_{CH} = \frac{h^2}{2\pi mk[v_H \zeta(3/2)]^{2/3}}$.

Using above equations, one can plot the curves of R_{QL} and R_e varying with the volume ratio r_v for given v_L , T_L and τ , as shown in Figs. 7 and 8, respectively. It is seen from the curves in Figs. 7(a) and 8(a) that, in order to satisfy the condition of $\Delta Q > 0$, the temperature ratio τ has to be chosen to be very large and that for given v_L , T_L and τ , both R_{QL} and R_e are monotonically decreasing

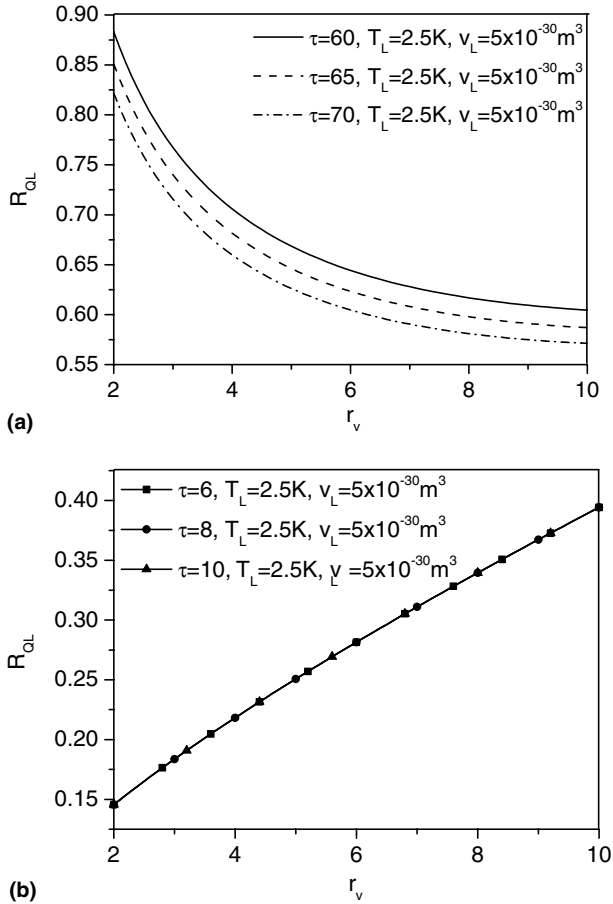


Fig. 7. The curves of R_{QL} varying with r_v in the temperature region of $T_H > T_c(v_L) > T_E > T_c(v_H) > T_L$ for some given values of v_L , T_L and τ . Curves are presented for (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$.

functions of r_v and the larger the temperature ratio τ , the smaller the R_{QL} (and R_e). This implies that the effect of the regenerative losses on the performance of the cycle is much larger than that of quantum degeneracy and, in such a case, the larger the volume ratio r_v , the larger the regenerative losses. Figs. 7(b) and 8(b) show that when $\Delta Q < 0$, both R_{QL} and R_e are monotonically increasing functions of r_v for given v_L , T_L and τ . This indicates that when $\Delta Q < 0$, the effect of quantum degeneracy on the performance of the cycle is much larger than that of regenerative losses and the larger the volume ratio r_v , the stronger the quantum degeneracy.

• When $T_c(v_L) > T_H > T_E > T_L > T_c(v_H)$, there are two cases $\Delta Q > 0$ and $\Delta Q < 0$. When $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT > \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$, $\Delta Q > 0$ and Eqs. (12)–(14) are, respectively, written as

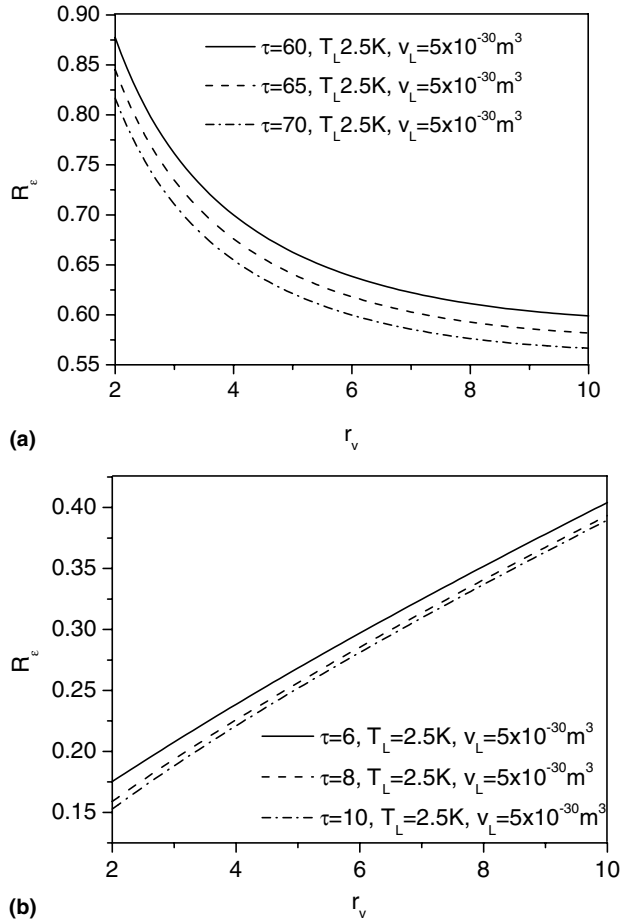


Fig. 8. The curves of R_e varying with r_v in the temperature region of $T_H > T_c(v_L) > T_E > T_c(v_H) > T_L$ for some given values of v_L , T_L and τ . Curves are presented for (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$.

$$\varepsilon = \frac{T_L \left[F(z_d) - 0.5134 \left(\frac{T_L}{T_{CL}} \right)^{3/2} - \ln z_d \right] + \frac{3}{2} T_H \left[F(z_a) - 0.5134 \left(\frac{T_H}{T_{CL}} \right)^{3/2} \right]}{T_H \left[F(z_a) - 0.5134 \left(\frac{T_H}{T_{CL}} \right)^{3/2} - \ln z_a \right] - T_L \left[F(z_d) - 0.5134 \left(\frac{T_L}{T_{CL}} \right)^{3/2} - \ln z_d \right]}, \quad (33)$$

$$R_{QL} = \frac{F(z_d) - 0.5134 (T_L/T_{CL})^{3/2} - \ln z_d + (3/2) \tau [F(z_a) - 0.5134 (T_H/T_{CL})^{3/2}]}{\ln(v_H/v_L)}, \quad (34)$$

and

$$R_e = \frac{(\tau - 1) \left\{ F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d + 1.5\tau[F(z_a) - 0.5134(T_H/T_{CL})^{3/2}] \right\}}{\tau[F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] + 0.5134(T_L/T_{CL})^{3/2} - F(z_d) + \ln(z_d)} \quad (35)$$

While $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT < \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$, $\Delta Q < 0$ and Eqs. (12)–(14) can be further simplified as

$$\varepsilon = \frac{T_L \left\{ (5/2)[F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d \right\}}{T_H[F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] - T_L[F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}, \quad (36)$$

$$R_{QL} = \frac{(5/2)[F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d}{\ln(v_H/v_L)}, \quad (37)$$

$$R_e = \frac{(\tau - 1) \left\{ (5/2)[F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d \right\}}{\tau[F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] - [F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}. \quad (38)$$

In this case, the variations of R_{QL} and R_e with r_v are shown in Figs. 9 and 10. It is clearly seen from Figs. 9 and 10 that, for the case of $\Delta Q > 0$, R_e is always smaller than unity, while R_{QL} is always larger than unity and both R_{QL} and R_e are monotonically decreasing functions of r_v ; while for $\Delta Q < 0$, both R_{QL} and R_e are monotonically increasing functions of r_v . It shows that, in this temperature order, the effect of quantum degeneracy of the working substance is distinct and the regenerative losses increase with the volume ratio r_v .

- When $T_c(v_L) > T_H > T_E > T_c(v_H) > T_L$, it is clearly seen from Fig. 2 that $\int_{T_E}^{T_H} (C_{VL} - C_{VH}) dT < \int_{T_L}^{T_E} (C_{VH} - C_{VL}) dT$. In this case, $\Delta Q < 0$ and Eqs. (12)–(14) can be, respectively, expressed as

$$\varepsilon = \frac{1.2835 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{T_H[F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] - 0.5134 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]},$$

$$R_{QL} = \frac{1.2835 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\ln(v_H/v_L)}, \quad (39)$$

and

$$R_e = \frac{1.2835(\tau - 1) [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\tau[F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] - 0.5134 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}. \quad (40)$$

- When $T_E > T_H > T_L > T_c(v_H)$ or $T_E > T_H > T_c(v_H) > T_L$, it can be seen, from Fig. 2, that $C_V(T, v_H) < C_V(T, v_L)$, so $\Delta Q < 0$. When $T_E > T_H > T_L > T_c(v_H)$, Eqs. (12)–(14) can be, respectively, expressed as

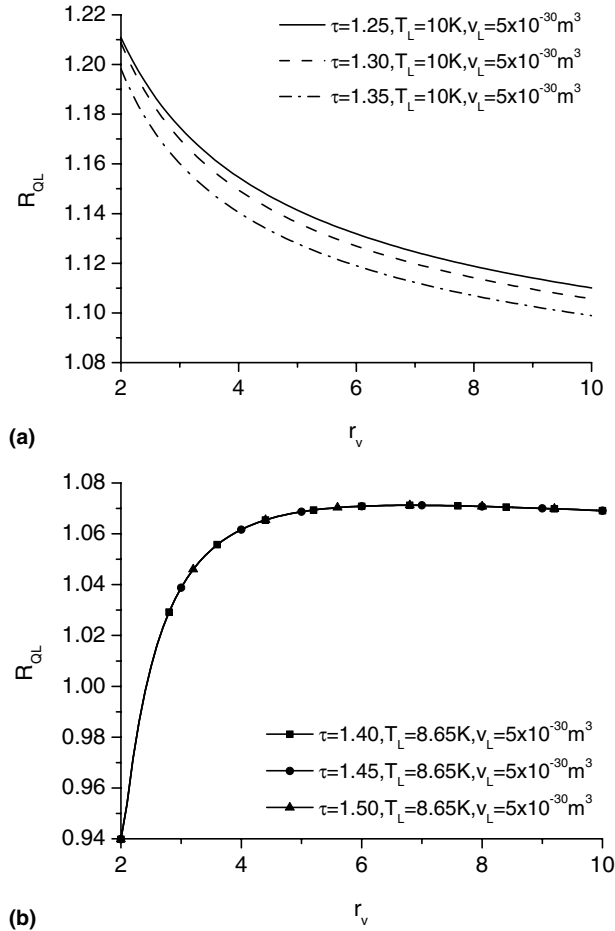


Fig. 9. The curves of R_{QL} varying with r_v in the temperature region of $T_c(v_L) > T_H > T_E > T_L > T_c(v_H)$ for some given values of v_L , T_L and τ . Curves are presented for (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$.

$$\varepsilon = \frac{T_L \left\{ (5/2) [F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d \right\}}{T_H [F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] - T_L [F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}, \quad (41)$$

$$R_{QL} = \frac{(5/2) [F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d}{\ln(v_H/v_L)}, \quad (42)$$

and

$$R_e = \frac{(\tau - 1) \left\{ (5/2) [F(z_d) - 0.5134(T_L/T_{CL})^{3/2}] - \ln z_d \right\}}{\tau [F(z_a) - 0.5134(T_H/T_{CL})^{3/2} - \ln z_a] - [F(z_d) - 0.5134(T_L/T_{CL})^{3/2} - \ln z_d]}. \quad (43)$$

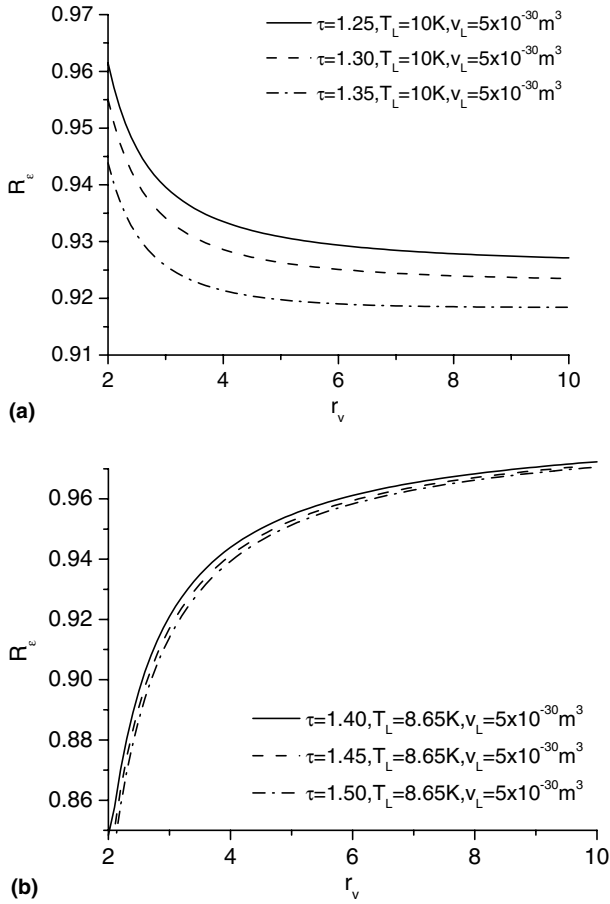


Fig. 10. The curves of R_e varying with r_v in the temperature region of $T_c(v_L) > T_H > T_E > T_L > T_c(v_H)$ for some given values of v_L , T_L and τ . Curves are presented for (a) $\Delta Q > 0$ and (b) $\Delta Q < 0$.

When $T_E > T_H > T_c(v_H) > T_L$, Eqs. (12)–(14) are, respectively, expressed as

$$\varepsilon = \frac{1.2835 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{T_H [F(z_a) - 0.5134 (T_H/T_{CL})^{3/2} - \ln z_a] - 0.5134 T_L [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}, \quad (44)$$

$$R_{QL} = \frac{1.2835 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\ln(v_H/v_L)}, \quad (45)$$

and

$$R_e = \frac{1.2835(\tau - 1) [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\tau [F(z_a) - 0.5134 (T_H/T_{CL})^{3/2} - \ln z_a] - 0.5134 [(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}. \quad (46)$$

In the last two cases, the variations of R_{QL} and R_e with r_v are shown in Figs. 11–16, respectively. It can be seen, from Figs. 11–16, that both R_{QL} and R_e are monotonically increasing functions of r_v and their values are smaller than unity. This indicates that in these temperature regions, the coefficient of performance and amount of refrigeration of the cycle are smaller than those of the classical Stirling refrigeration cycle and the regenerative losses and amount of refrigeration increase with the volume ratio r_v .

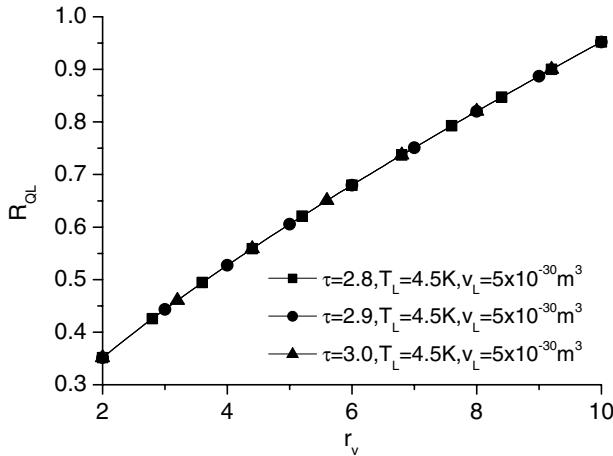


Fig. 11. The curves of R_{QL} varying with r_v in the temperature region of $T_c(v_L) > T_H > T_E > T_c(v_H) > T_L$ for some given values of v_L , T_L and τ .

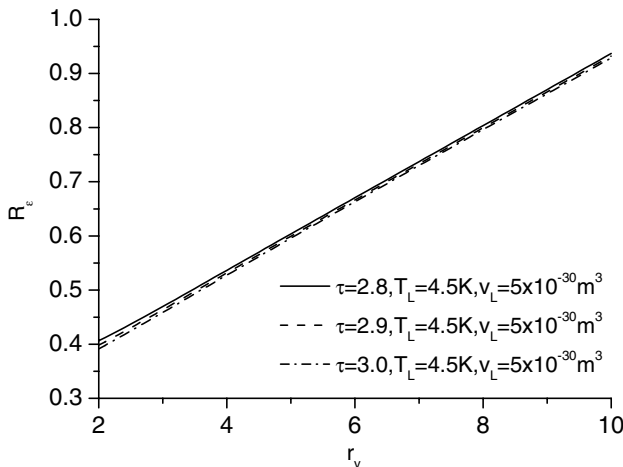


Fig. 12. The curves of R_e varying with r_v in the temperature region of $T_c(v_L) > T_H > T_E > T_c(v_H) > T_L$ for some given values of v_L , T_L and τ .

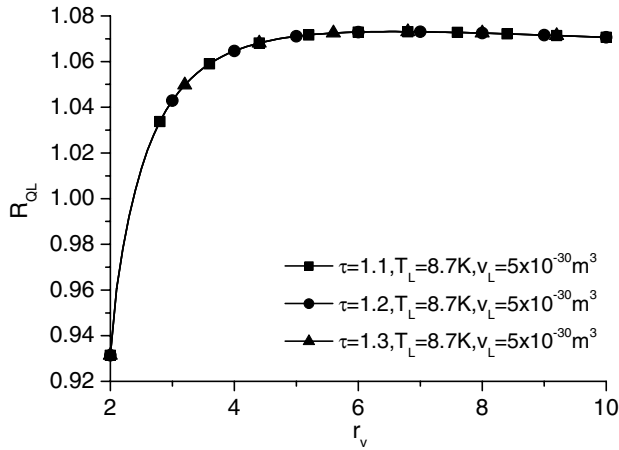


Fig. 13. The curves of R_{QL} varying with r_v in the temperature region of $T_E > T_H > T_L > T_c(v_H)$ for some given values of v_L , T_L and τ .

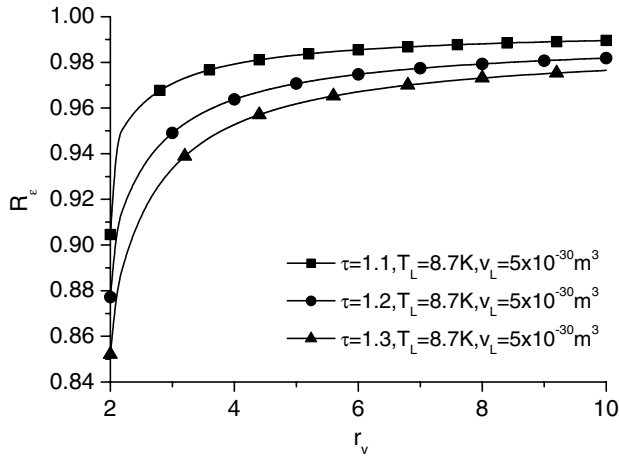


Fig. 14. The curves of R_e varying with r_v in the temperature region of $T_E > T_H > T_L > T_c(v_H)$ for some given values of v_L , T_L and τ .

• When $T_c(v_c) > T_E > T_c(v_H) > T_H > T_L$, it is clearly seen from Fig. 2 that $C_V(T, v_H) > C_V(T, v_L)$, so $\Delta Q < 0$. In this case, Eqs. (12)–(14) can be simplified as

$$\varepsilon = \frac{5/2}{\tau^{5/2} - 1}, \quad (47)$$

$$R_{QL} = \frac{(5/2)[(T_L/T_{CH})^{3/2} - (T_L/T_{CL})^{3/2}]}{\ln(v_H/v_L)}, \quad (48)$$

$$R_e = \frac{5(\tau - 1)}{2(\tau^{5/2} - 1)}. \quad (49)$$

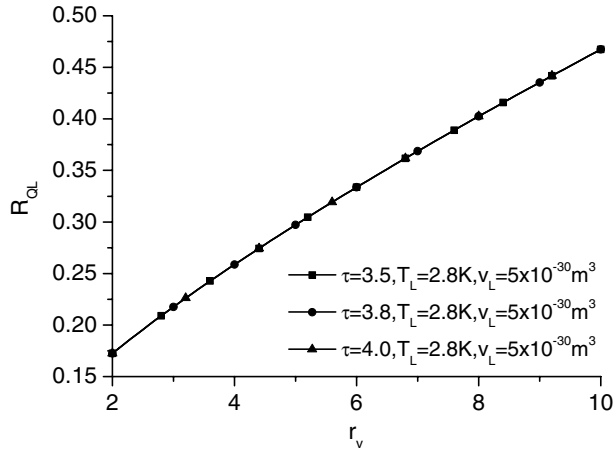


Fig. 15. The curves of R_{OL} varying with r_v in the temperature region of $T_E > T_H > T_c(v_H) > T_L$ for some given values of v_L , T_L and τ .

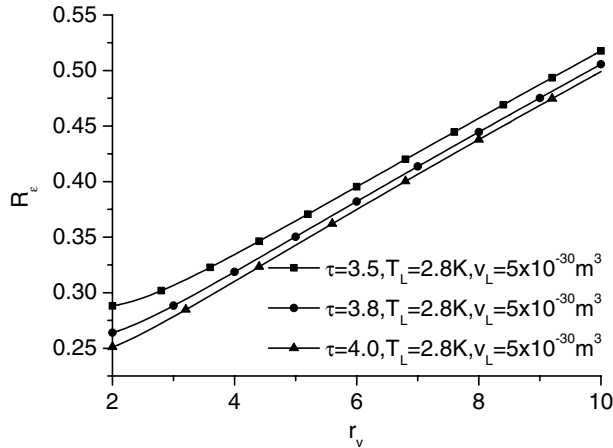


Fig. 16. The curves of R_e varying with r_v in the temperature region of $T_E > T_H > T_c(v_H) > T_L$ for some given values of v_L , T_L and τ .

• When the temperature of the gas is high enough and its density is low enough, the fugacity of the Bose gas z is much smaller than unity. In such a case, $g_A(z) = z$, $F(z) = 1$, and an ideal Bose gas becomes an ideal classical gas. Eqs. (9), (10), (12), (13) and (14) can be, respectively, simplified as $\Delta Q = 0$, $Q'_L = Q_L^c$, $\varepsilon = \varepsilon_c$, $R_e = 1$ and $R_{OL} = 1$. The results are just those of a classical Stirling refrigeration cycle. This shows clearly that, at high temperatures, the Bose-Stirling refrigeration cycle becomes the classical Stirling refrigeration cycle.

6. Conclusions

It is clearly seen from the results obtained above that because there are inherent regenerative losses, the Stirling refrigeration cycle working with an ideal Bose gas may not possess the condition of perfect regeneration. On the other hand, due to the existence of BEC, the performance of the Stirling refrigeration cycle operated in the different temperature regions is obviously unique. The influences of the inherent regenerative losses and the quantum degeneracy on the coefficient of performance and amount of refrigeration of the cycle have been analyzed in detail. The coefficients of performance of the Bose-Stirling refrigeration cycle, under different conditions, are derived analytically. In general, the coefficient of performance depends not only on the temperatures of two heat-reservoirs but also on the volumes of two constant-volume processes and other parameters, and are always lower than that of a Stirling refrigeration cycle working with a classical ideal gas. Only if the gas temperature is low or high enough, can the coefficient of performance be a function of temperature, but independent of other parameters. The results obtained here reveal the general performance characteristics of the Bose-Stirling refrigeration cycle operated in the various different temperature regions.

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